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Quantum Error Correction in Silicon Charge Qubits

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Abstract—The interaction of the quantum register with a noisy environment that leads to phase and bit errors is considered. Modeling of 5-qubit and 9-qubit error-correction algorithms for various environments is performed. It is shown that the use of the quantum correction leads to a quadratic decrease in the error probability. The efficiency of applying the 5-qubit algorithm of error correction for a silicon double-dot qubit is shown.

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INTRODUCTION

One of the main problems that stand in the way of designing a full-fledged quantum computer is decoherence of the quantum system [1, 2]. Because of the contact with the environment, the quantum register is no longer in the pure state described by wave function ψ. The register transforms into a mixed state described by density matrix ρ [3–5]. Thus, as a result of the interaction of the qubit with the surrounding, distortions of their quantum state appear. In order to provide reliable operation of a quantum computer, it is necessary to find the method for preventing these errors, for example [6, 7], or correcting these errors. The theory of quantum error correction [8–15] is an important element of the quantum information theory. The method of correction of the errors makes it possible to restore the state of the quantum system after the noise effect.

In this study, we will consider the correcting circuits for the errors [16, 17] in the silicon double-dot qubit. The task of correcting circuits is to increase the storage time (or the probability of a transfer along a noisy channel) of a one-qubit state described by density matrix ρ_{in} (the Hermitian positive operator with a unity trace). We will divide the process of error correction into three stages, namely, coding, the effect of noises, and decoding (Fig. 1), assuming that the coding and decoding procedures are ideal. Let us consider each stage separately.

The addition of a certain number of additive qubits (ancillas) to protect a message against noise occurs at the coding stage. Thus, even if the information in the coded message is damaged, the excess will allow us to restore the entire starting information during the message decoding. During coding, state ρ_{in} is coupled with additional qubits (ancillas), which are initially in state $|0\rangle\langle 0|$, with the help of quantum gates. The transformation is unitary, i.e.,

$$\rho_{\text{in}} \otimes |0\rangle\langle 0| \otimes |0\rangle\langle 0| \to U(\rho_{\text{in}} \otimes |0\rangle\langle 0| \otimes |0\rangle\langle 0|)U^{\dagger}.$$

At the stage of the noise effect, a quantum register comes to contact with the environment. We will describe the evolution of such a qubit system using quantum operation [5] (quantum channel [18, 19] and superoperator [20]) $\mathbf{S}$, i.e., $\rho \to \mathbf{S}\rho$. The quantum operation over a system of n qubits is a linear reflection $\mathbf{S} : H_{2^n} \to H_{2^n}$, which preserves the trace (the Schrödinger pattern) and is quite positive [18, 19].

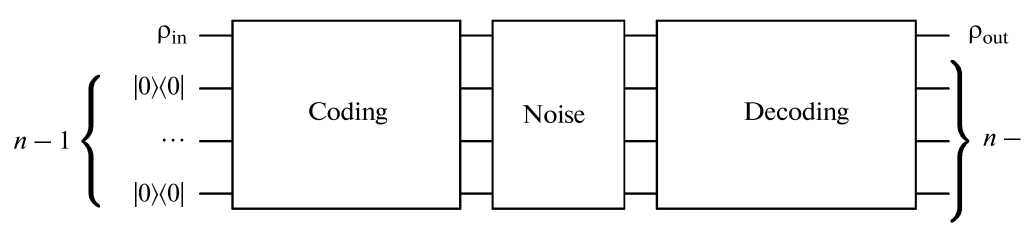

Fig. 1. Stages of the error correction process.

Linear mapping $\mathbf{S} : H_d \to H_d$ is quantum operation [21] if and only if $\exists E_1, E_2, \ldots, E_n \subseteq H_d, \sum_i E_i^\dagger E_i = I$;

$$\mathbf{S}\rho = \sum_i E_i \rho E_i^\dagger, \tag{1}$$

i.e., the noise effect has the representation of the operator sum (the Kraus representation [22]).

During decoding, the damaged quantum states are analyzed, and some transformations are performed over the system based on the results of the analysis. It was shown [5] that a principle of selection by the majority from the classical information theory can be used as such an analysis.

To evaluate the quality of the correction, we will use the notion of measure of decoherence D [23]. By definition, the measure of decoherence is the operator norm for operator $\rho_{out} - \rho_{in}$, where ρ_{out} is the qubit state after the noise effect and correction procedure, which is maximal over all the states of the system. The operator norm of the Hermitian operator A is determined as $\|A\| = \max_{a \in \mathrm{spec}(A)}|a|$ [18, 19]; here, spec(A) is the spector of operator A. It was shown [23] that the maximum of the operator norm is attained on pure states. For the procedure without the error correction,

$$D = \sup_{\theta,\varphi}\left\|\rho_{out}^{(1)} - \rho_{in}^{(1)}\right\|, \tag{2}$$

where θ and φ are the parameters which completely describe the pure state. Taking the larger number of the states from the Bloch sphere, we can attain the sufficiently exact measure of decoherence during the numerical modeling.

MODELING OF VARIOUS TYPES OF ERRORS

Let us assume that we want to transmit the information a qubit along the channel with a bit (classical) error. Let the environment change $|0\rangle$ to $|1\rangle$ with probability $p > 0$, and assume the qubit is transmitted without the error with probability $1 - p$. In a classical information theory, such a channel corresponds to a binary symmetric channel [5]. It is convenient to write this action using the Kraus representation (1). It is convenient to select the Pauli matrices $\{I, X, Y, Z\}$ as the basis $\{E_\alpha\}_{\alpha=0}^{d^2-1}$ in space H_d for $d = 2$ (one qubit). In this case, the operator sum in the case under consideration is written as

$$\mathbf{S}\rho^{(1)} = (1-p)I\rho^{(1)}I + pX\rho^{(1)}X. \tag{3}$$

We note that in the case of error (3), the measure of decoherence is $D = \sup_{\theta,\varphi}\|\mathbf{S}\rho^{(1)} - \rho^{(1)}\| = p$. Therefore, $0 \le p \le 1$ is the decoherence probability of one qubit. Density matrix $\rho^{(k)}$ describes the quantum register of k qubits.

Let us consider three qubits. It is convenient to model the environment in the case of the independent action on qubits using the sequential application of (3):

$$\begin{aligned}\mathbf{S}\rho^{(3)} &= (1-p)^3 I\rho^{(3)}I + (1-p)^2 pX_1\rho^{(3)}X_1 + (1-p)^2 pX_2\rho^{(3)}X_2 + (1-p)^2 pX_3\rho^{(3)}X_3 \\ &+ (1-p)p^2 X_1X_2\rho^{(3)}X_2X_1 + (1-p)p^2X_1X_3\rho^{(3)}X_3X_1 + (1-p)p^2X_2X_3\rho^{(3)}X_3X_2 + p^3X_1X_2X_3\rho^{(3)}X_3X_2X_1.\end{aligned} \tag{4}$$

Operator X_i is the Pauli operator X affecting the ith qubit.

Let us consider the type of the correction scheme of a bit error in which only unitary operators are present [24]. To correct the bit error, we used the experience of error correction in the classic information. The bit error can be corrected using coding by three qubits:

$$\begin{aligned}|0\rangle &\to |000\rangle, \\ |1\rangle &\to |111\rangle.\end{aligned} \tag{5}$$

Despite the error, it is possible to find the correct result via decoding. Let us consider the decoding procedure: during the inversion of the first qubit, all qubits are inverted, and when inverting each of the two other qubits, only those two are inverted. To correct the first qubit, it is necessary to invert it only if the other two equal $|1\rangle$. The circuit correcting the bit errors can be presented in Fig. 2. By the results of modeling (Fig. 3) or the correction process of classical error (4), we established that the measure of decoherence coincides with the measure of decoherence in the case of a binary symmetric channel and equals

$$D = 3p^2(1-p) + p^3 = 3p^2 - 2p^3.$$

It should be noted that noise increases the system's entropy. In this case, error correction decreases the entropy. In the case of the circuit with syndromes, the entropy decreases during the measurement and is invariable during rejection of two undesirable qubits. In the case of the circuit without measurements, the entropy decreases during the rejection of the additional qubits.

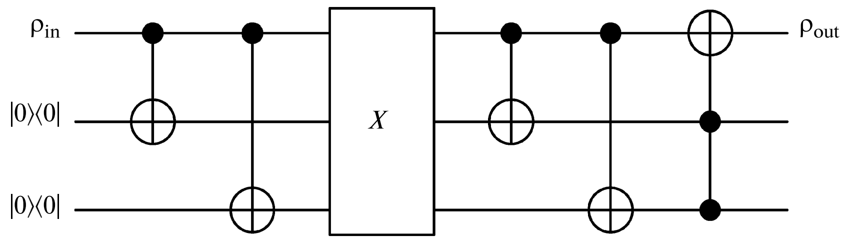

Fig. 2. Bit error correction circuit.

It was shown [16] that the code is able to correct any quantum error if it is able to correct the phase and bit errors. The bit error is the unitary transformation $\rho \to X\rho X^\dagger$, and the phase error is transformation $\rho \to Z\rho Z^\dagger$. The correction of the phase error differs from the correction of the bit error only by the basis, in which the environment affects the qubits. It is possible to construct the correction code without correction procedures [25] based on the previously suggested 9-qubit code [16] (Fig. 4).

It was established that the minimal number of qubits necessary to correct the arbitrary error in one qubit equals five. The example of the 5-qubit correction code is the DiVincenzo–Shor code [17]. Based on this code, the circuit using only unitary operators during correction was constructed [25] (Fig. 5). Operator R, which is applied in the circuit, can be written in matrix form:

$$R \equiv \begin{pmatrix} |0\rangle_L^T \\ (W_1|0\rangle_L)^T \\ (W_2|0\rangle_L)^T \\ \dots \\ (W_{15}|0\rangle_L)^T \\ |1\rangle_L^T \\ (W_1|1\rangle_L)^T \\ (W_2|1\rangle_L)^T \\ \dots \\ (W_{15}|1\rangle_L)^T \end{pmatrix}. \quad (6)$$

Vector W determines all possible errors $\{I, X_1, X_2, \dots, X_5, Y_1, Y_2, \dots, Y_5, Z_1, Z_2, \dots, Z_5\}$.

We will consider the arbitrary error by the example of the depolarizing channel [5]. We write the operator sum in the form

$$\mathbf{S}\rho^{(1)} = \left(1 - \frac{3}{2}p\right) I\rho^{(1)}I + \frac{p}{2}(X\rho^{(1)}X + Y\rho^{(1)}Y + Z\rho^{(1)}Z). \quad (7)$$

Here, $0 \le p \le \frac{2}{3}$ is the probability of decoherence of each qubit. The results of correction using the Shor and DiVincenzo–Shor algorithms are presented in Fig. 6.

QUBIT AS A DOUBLE QUANTUM DOT

Double quantum dots were introduced as a promising element of quantum computers in [26]. A double quantum dot consists of two quantum dots with one electron associated in a tunnel manner. The two lower intrinsic quantum states are the symmetric and asymmetric ones. The main advantage of a double quantum dot is that we can attain a narrow energy band between these states using a decrease in the tunnel coupling. It was shown that this leads to a substantial suppression of the interaction of the qubit with phonons [27, 28].

The dots themselves are cut from a two-dimensional electron gas and are controlled by the field at the gates. Electrons in the points interact with each other only through the direct Coulomb interaction. The exchange interaction is absent since there is no overlapping of wave functions. The possibility of fulfilling some quantum logic operations using such qubits was proven, which opens the prospects for developing a computer with a large number of qubits.

We consider the electron in a double well potential. Such a structure can be fabricated as two quantum dots, whose geometry is determined by external metal gates and the electric potential across them. The structure can be also fabricated as two coupled close phosphor donors injected into silicon. As a result, we obtain a qubit with basis states $|0\rangle$ and $|1\rangle$, which describe the electron localized near the left and right minima of the potential, respectively. We assume that the temperature is sufficiently low in order that no transitions to higher

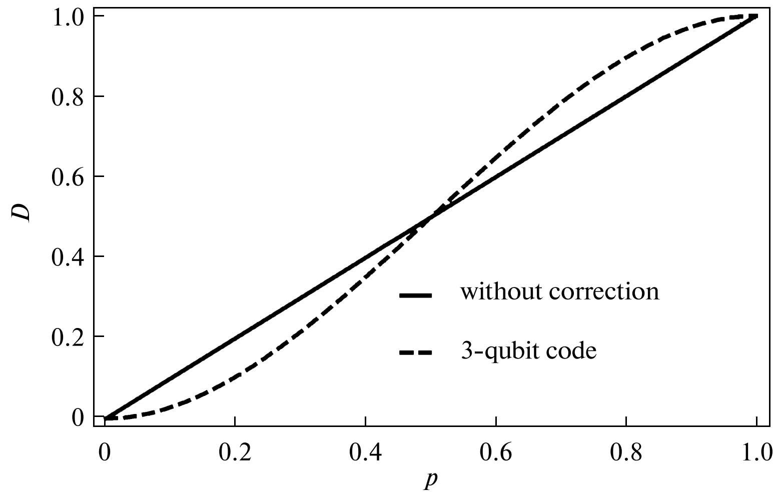

Fig. 3. Decoherence in the case of bit and phase errors.

levels would occur. Decoherence in such a system occurs due to the interaction with acoustic phonons.

The total Hamiltonian of the electron and the phonon surrounding [27] is

$$H = H_e + H_p + H_{ep}. \tag{8}$$

$$H_e = -\frac{1}{2}\varepsilon_A(t)X - \frac{1}{2}\varepsilon_P(t)Z. \tag{9}$$

The electron Hamiltonian (9) is expressed through parameters ε_A and ε_P, which can be controlled by means of external metal gates. Parameter ε_A determines the height of the hump between the potential minima, and parameter ε_P determines the difference between these minima. These parameters determine splitting ε between the ground state of the electron and the first excited state in the energy basis. This splitting has the form $\varepsilon = \sqrt{\varepsilon_A^2 + \varepsilon_P^2}$. The Hamiltonian of the phonon component is written in the form

$$H_p = \sum_{q,\lambda} \hbar s q b_{q,\lambda}^\dagger b_{q,\lambda}, \tag{10}$$

where $b_{q,\lambda}^\dagger$ and $b_{q,\lambda}$ are the operators of the birth and annihilation of phonons with wave number q and polarization λ. For simplicity, we consider the isotropic acoustic phonons with a linear dispersion law. The electron–phonon interaction is described by the Hamiltonian

$$H_{ep} = \sum_{q,\lambda} Z(g_{q,\lambda} b_{q,\lambda}^\dagger + g_{q,\lambda}^* b_{q,\lambda}), \tag{11}$$

where $g_{q,\lambda}$ is the coupling constant, which depends on the specific system configuration and interaction type.

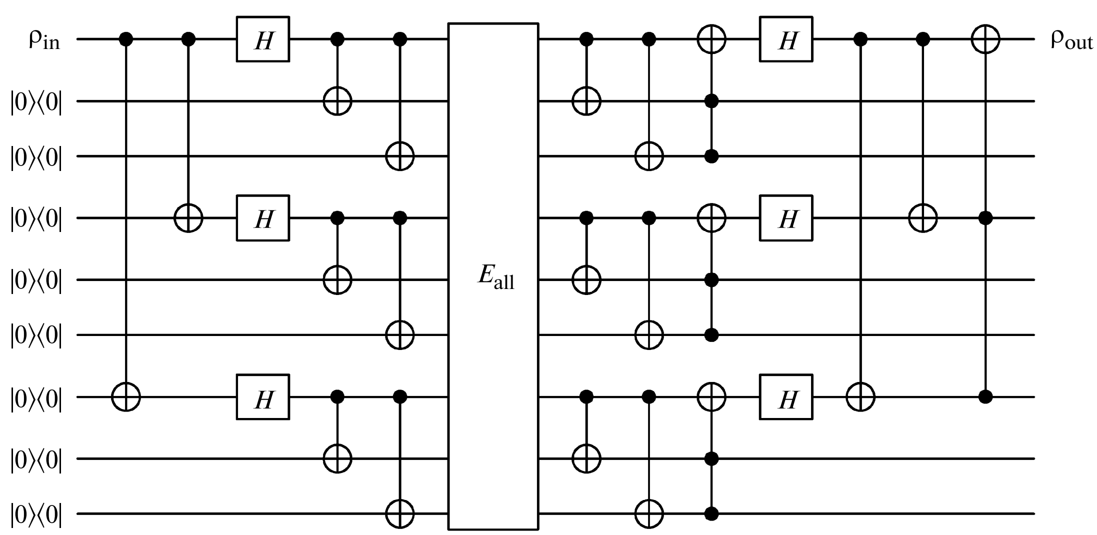

Fig. 4. Circuit of the 9-qubit Shor code.

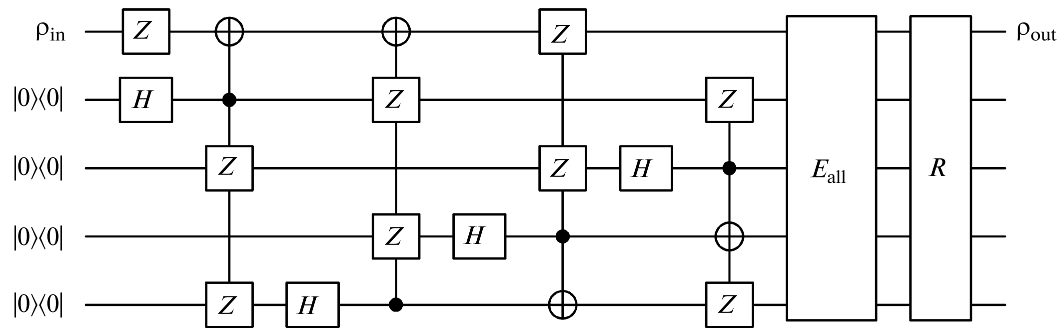

Fig. 5. Circuit of the 5-qubit DiVincenzo–Shor code.

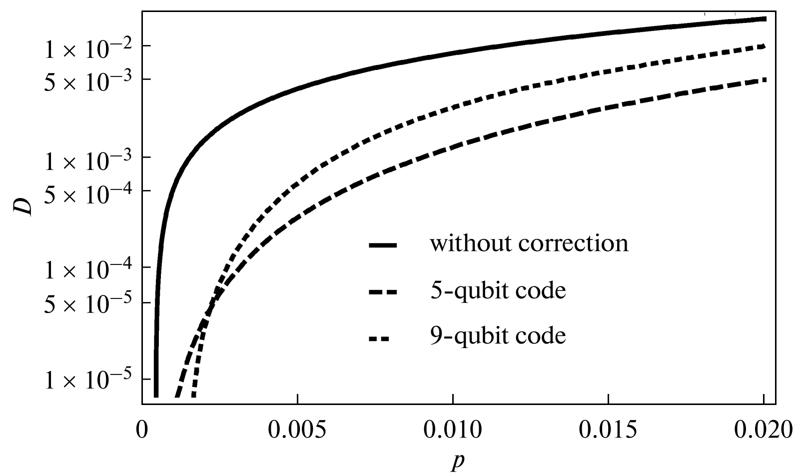

Fig. 6. Decoherence in the case of the depolarizing channel.

The distance between the quantum dots $L = 50$ nm and their finite size $a = 3$ nm decrease the effect of the electron–phonon interaction at the ends of the phonon spectrum.

Interaction (11) leads to decoherence of the qubit. The coherence loss is a certain function of parameters ε_A and ε_P.

Let us consider the qubit at zero temperature. The main parameter of the dots, which affects the interaction with phonons, is their size a. Let us consider the dots of the Gaussian shape, in which the wave function is the Gaussian $\Psi(r) \sim \exp(-r^2/(2a^2))$, since the remote electrons form the parabolic potential. We will assume that the phonon wavelength is sufficiently large compared to the dot size a and the distance between the dots, i.e., $ak \ll 1$ and $Lk \ll 1$. Using these assumptions, we can find relaxation rate $\Gamma(\varepsilon)$ due to the interaction with deformation phonons:

$$\Gamma(\varepsilon) = \frac{\Xi^2 L^2 \varepsilon^5}{24\pi\rho s^7 \hbar^6}, \tag{12}$$

where $k = \varepsilon/(s\hbar)$ is the wave vector of the emitted phonon. In the numerical calculations, we used deformation potential $\Xi = 3.3$ eV, semiconductor density $\rho = 2.33$ g/cm^3, and velocity of sound $s = 9 \times 10^3$ m/s.

Let us consider the basis

$$|+\rangle = \frac{1}{\sqrt{2}}[|0\rangle + |1\rangle], \quad |-\rangle = \frac{1}{\sqrt{2}}[|0\rangle - |1\rangle].$$

In this basis, the evolution of the density matrix can be written as

$$\rho \to \begin{pmatrix} 1 - \rho_{--}(0)e^{-\Gamma(\Delta t)\Delta t} & \rho_{+-}(0)e^{-\Gamma(\Delta t)\Delta t/2 + i\varepsilon\Delta t/\hbar} \\ \rho_{-+}(0)e^{-\Gamma(\Delta t)\Delta t/2 - i\varepsilon\Delta t/\hbar} & \rho_{--}(0)e^{-\Gamma(\Delta t)\Delta t} \end{pmatrix}. \tag{13}$$

Splitting ε does not affect the error level, since we specify such time $\Delta t = \pi\hbar/\varepsilon$ for the fulfillment of calculations that multiplier $e^{i\varepsilon\Delta t/\hbar} = -1$ would accept the value required by us and operation *NOT* would be fulfilled.

To describe the interaction of the qubit system with phonons, we should use the formalism of superoperators. Using representation (13) and assuming $\varepsilon = 0$, we can find superoperator **S**:

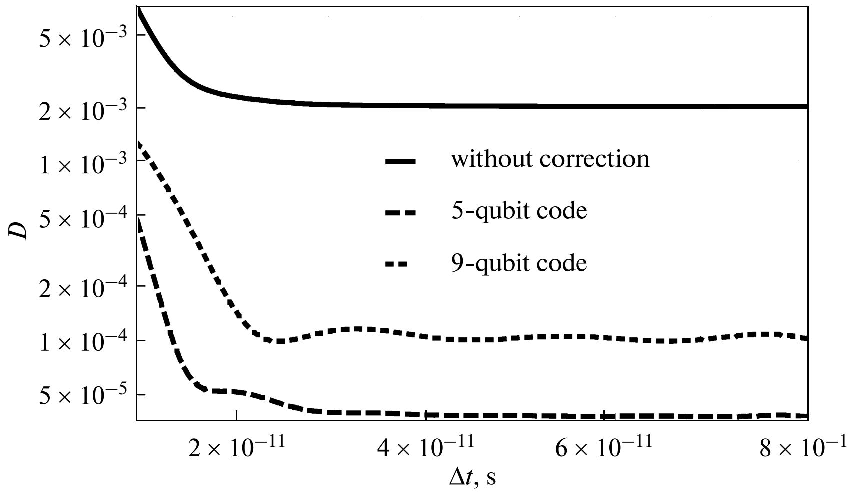

Fig. 7. Decoherence after 68 operations in the double quantum dot qubit.

$$\mathbf{S}\rho = \begin{pmatrix} 1 & 0 \\ 0 & e^{-\Gamma\Delta t/2} \end{pmatrix} \rho \begin{pmatrix} 1 & 0 \\ 0 & e^{-\Gamma\Delta t/2} \end{pmatrix}^{\dagger} + \begin{pmatrix} 0 & \sqrt{1-e^{-\Gamma\Delta t}} \\ 0 & 0 \end{pmatrix} \rho \begin{pmatrix} 0 & \sqrt{1-e^{-\Gamma\Delta t}} \\ 0 & 0 \end{pmatrix}^{\dagger}. \quad (14)$$

According to [27], the measure of decoherence is $D = 1 - e^{\Gamma\Delta t}$.

Phase damping occurs in the system. The evolution of the density matrix for one operation could be presented in the form

$$\rho \to \begin{pmatrix} \rho_{00}(0) & \rho_{01}(0)e^{-B^2(\Delta t)+i\varepsilon\Delta t/\hbar} \\ \rho_{10}(0)e^{-B^2(\Delta t)-i\varepsilon\Delta t/\hbar} & \rho_{11}(0) \end{pmatrix}. \quad (15)$$

Using representation (15) and assuming $\varepsilon = 0$, we can find superoperator $\mathbf{S}$:

$$\mathbf{S}\rho = \begin{pmatrix} e^{-B^2(\Delta t)/2} & 0 \\ 0 & e^{-B^2(\Delta t)/2} \end{pmatrix} \rho \begin{pmatrix} e^{-B^2(\Delta t)/2} & 0 \\ 0 & e^{-B^2(\Delta t)/2} \end{pmatrix}^{\dagger}$$
$$+ \begin{pmatrix} \sqrt{1-e^{-B^2(\Delta t)}} & 0 \\ 0 & 0 \end{pmatrix} \rho \begin{pmatrix} \sqrt{1-e^{-B^2(\Delta t)}} & 0 \\ 0 & 0 \end{pmatrix}^{\dagger} + \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{1-e^{-B^2(\Delta t)}} \end{pmatrix} \rho \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{1-e^{-B^2(\Delta t)}} \end{pmatrix}^{\dagger}. \quad (16)$$

According to [14], the measure of decoherence is $D = (1 - e^{-B^2(\Delta t)})/2$. Here, $B^2(t)$ is the spectral function having the form (for the deformation interaction) [27]:

$$B^2(t) = \frac{\Xi^2}{\pi^2\hbar\rho s^3}\int_0^{\infty} q^2 dq \int_0^{\pi} \sin\Theta d\Theta \frac{\sin^2(qL\cos\Theta)\exp(-a^2q^2/2)}{q}\sin^2\frac{qst}{2}. \quad (17)$$

During the calculations, transmitting, or storing the information, the damping of amplitude (13) and phase (15) occur. To fight against this decoherence process, we can use algorithms of error correction which were considered earlier in this study.

Let us consider error correction in silicon in the case of the electron state in a double quantum dot. To determine the relaxation rate and the spectral function, we will use formulas (12) and (17). We will take into account the fact that the errors caused by the amplitude and phase relaxation act for time $t = n\Delta t$. To describe the entire system of five or nine qubits, we should use superoperators (14) and (16), with n-fold application of superoperators during n operations. The results of modeling the 5-qubit DiVincenzo–Shor processes as applied to error correction in silicon for the qubit based on the double quantum dot are presented in Fig. 7 (for $n = 68$). Here, we assume that correction circuits are ideal and no errors occur during the correction. In this case, in the limits of all operations $D < 10^{-4}$ for the corrected qubit, which satisfies the condition of stability of the calculations for the errors.

CONCLUSIONS

We considered the interaction of the quantum register with the environment leading to various errors. We analyzed the channels of the bit and phase errors, damping of the amplitude and the phase, and the depolarizing channel. Quantum codes of error correction, which are necessary to perform the full-fledged quantum calculations, are considered. The possibility of using the quantum correction codes without measurement operations was shown, and modifications of the Shor and DiVincenzo–Shor correcting circuits, in which the determination of syndromes of the errors is absent, was analyzed. Modeling of the 5-qubit and 9-qubit algorithms of error correction for various cases of the environment's action, namely, the depolarizing channel and the channel of damping the amplitude and the phase, is performed. The disadvantage of the 5-qubit algorithm is the lack of a sufficiently simple representation of the decoding circuit. It is shown by the example of the bit and phase errors that the use of quantum correction at low noise levels leads to the quadratic representation of the electron state in a double quantum well. The efficiency of applying the 5-qubit error correction algorithm for the silicon double-dot qubit is established.

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